

Channel Capacity of Adaptive Transmission Techniques over Rice (Nakagami- n) Fading Channels

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Abstract—Expression of channel capacity for various adaptive rate and power transmission techniques have been derived over Nakagami- n (Rice) fading channels. A probability density function based approach has been adopted to find out the mathematical expressions for different possible rate and power adaptation techniques. The derived expressions are numerically evaluated for different fading conditions and study on the effect of fading parameters on the channel capacity with different adaptive transmission schemes has been presented.

Index Terms: Channel capacity, Rice (Nakagami- n) fading channel, adaptive transmission technique, quality of service.

1. INTRODUCTION

Capacity analysis over multipath fading channels provides very useful information to the design engineers to design and study of modern wireless communication systems. Information of channel capacity also helps the system engineer to provide quality of service in an environment with limited available spectrum and huge number of users. Multipath propagation of the wireless signals leads to the fading effect and due to which the performance of the wireless receivers degrades. However, in last few decades, researchers are able to mathematically model the fading effect in different propagation environment, which in turn helps in designing and implementation of an efficient wireless communication system. Rice (Nakagami- n) fading is used to model propagation paths consisting of one strong direct line of sight component and many random weaker components [1]. Many research works analyze the channel capacity of various fading channels with and without diversity [2]- [11]. In [11], although capacity of rice fading channels have been derived for maximal ratio diversity combining the information of capacity with adaptive transmission techniques are missing. This generates a motive to know the capacity information of Rice fading channels with adaptive transmission techniques. A detailed study of the effect of fading parameter on the channel

capacity and comparison of different transmission techniques has been presented.

The rest of this paper is organized as follows. In Section II, we introduce the channel and system model. Different capacity formulas available in literature are discussed in Section III. In Section IV, capacity of adaptive system have been derived for different transmission technique and in Section V, we present numerical results. Finally, the paper is concluded in Section VI.

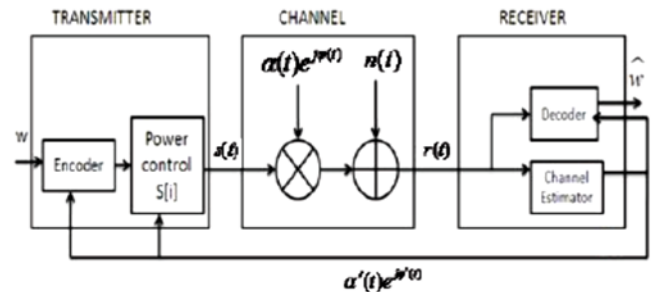


Fig. 1: System Model for Adaptive Transmission Technique

2. CHANNEL AND SYSTEM MODEL

We consider a cellular communication system receiving multipath fading signals in the presence of additive white Gaussian noise (AWGN). The channel is assumed to be slow, frequency nonselective with Rice fading statistics. The complex low pass equivalent of the received signal over one bit duration T_b can be expressed as

$$r(t) = \alpha e^{j\phi} s(t) + n(t), \quad 0 \leq t \leq T_b, \quad (1)$$

Where $s(t)$ is the transmitted bit with energy E_b and $n(t)$ is the complex Gaussian noise having non-zero mean and variance σ^2 two sided power spectral density $2N_0$. Random variable

(RV) φ represents the phase and α is the Rice distributed fading amplitude whose PDF can be given by [1]

$$p(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} \exp\left[\frac{-(1+n^2)\alpha^2}{\Omega}\right] I_0\left(2n\alpha\sqrt{\frac{1+n^2}{\Omega}}\right) \quad (2)$$

where $\Omega_t = E[\alpha_t^2]$, n is the Rice fading parameter and $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order. The system model is shown in Fig. 1, which transmit the message w from one end to the other through wireless fading channels. The message is encoded into the code words $s(t)$, which is then transmitted over the wireless channels. The transmitted signal experiences fading during its propagation, and it is modified with a gain $\alpha(t)e^{j\varphi(t)}$ in addition to the white Gaussian noise. We assume that the receiver is perfectly estimating the channel gain from the receive signal $r(t)$ and send back to the transmitter through a lossless path. From the information of the channel gain the transmitter adapts the power and rate of transmission according to transmission scheme used.

3. CAPACITY OF RICE FADING CHANNEL

3.1 Capacity Formulas

Channel capacity has been analyzed for various adaptive transmission schemes in last few decades. Analytical expressions for capacities based on different transmission techniques have been presented in [2] and [10]. In this analysis we use these formulas to obtain capacity expressions of the receiver over Rice fading channels. The formulas for different schemes are presented below for convenience.

1) Optimal Power and Rate Adaptation at the Transmitter:

For a system with a constraint on the average transmitting power, using optimal power and rate adaptation (OPRA) technique at the transmitter the channel capacity (bits/s) is given by [2]

$$C_{opra} = B \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_{\gamma}(\gamma) d\gamma \quad (3)$$

Where B is the channel bandwidth, $f_{\gamma}(\gamma)$ is the PDF of the output SNR and γ_0 is the optimal cut-off SNR, below which no transmission is allowed, has to satisfy the condition.

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_{\gamma}(\gamma) d\gamma = 1 \quad (4)$$

2) Constant Transmitting Power: When the transmitting power of the system is constant and optimal rate adaptation (ORA) technique is used at the transmitter, the channel capacity (bits/s) can be given by [2]

$$C_{ora} = B \int_0^{\infty} \log_2(1+\gamma) f_{\gamma}(\gamma) d\gamma \quad (5)$$

3) Channel Inversion with Fixed Rate: When the transmitter adapts its power to maintain constant received SNR so that inversion of the channel fading effects is possible, the system is said to be adopting channel inversion with fixed rate (CIFR) technique. The channel capacity (bits/s) for this case is given by [2]

$$C_{cifr} = B \log_2\left(1 + \frac{1}{R_{cifr}}\right) \quad (6)$$

Where

$$R_{cifr} \square \int_0^{\infty} \frac{1}{\gamma} f_{\gamma}(\gamma) d(\gamma)$$

4) Truncated Channel Inversion with Fixed Rate: This is a modified version of CIFR. When the channel goes into deep fades, to maintain constant receiver SNR a large amount of power is required at transmitter. So, to overcome this problem truncated channel inversion with fixed rate (TIFR) method is employed. In this case, the channel inversion is done when the receiver SNR is above a threshold value γ_0 . The capacity formula for TIFR can be given by [10]

$$C_{tifr} = B \log_2\left(1 + \frac{1}{R_{tifr}}\right) [1 - P_{out}(\gamma_0)] \quad (7)$$

where $R_{tifr} \square \int_{\gamma_0}^{\infty} \frac{1}{\gamma} f_{\gamma}(\gamma) d(\gamma)$ $P_{out}(\gamma_0) = \int_0^{\gamma_0} f_{\gamma}(\gamma) d(\gamma)$

It can be observed from the expressions in (3)-(7) that an analysis of the capacity of a system requires an expression for the PDF of the system output SNR i.e., $f_{\gamma}(\gamma)$. We present the capacity analysis in the following section using the PDF given in [1].

4. CAPACITY OF ADAPTIVE SYSTEM

The PDF of SNR for rice fading channel can be obtained from (2) by performing random variable (RV) transformation corresponding to taking the square of the RV followed by multiplying by a factor E_b/N_0 . Which is given in [1] as

$$f_{\gamma}(\gamma) = \frac{(1+n^2)}{\gamma} e^{-n^2} \exp\left(-\frac{(1+n^2)}{\bar{\gamma}}\gamma\right) I_0\left(2n\sqrt{\frac{(1+n^2)}{\bar{\gamma}}}\gamma\right) \quad (8)$$

A. Optimal power and rate adaptation at the transmitter

Putting (8) into (3), and solving the above integral using the formula $J_n(\mu) = \int_1^\infty t^{n-1} \ln(t) e^{-\mu t} dt$ for $\mu > 0$ which is for

integer n , $J_n(\mu) = \frac{(n-1)!}{\mu^n} \sum_{k=0}^{n-1} \frac{\Gamma(k, \mu)}{k!}$ [1], the capacity for opora scheme can be given as

$$C_{opra} = \frac{B e^{-n^2} \log_2 e}{\gamma_0^2} \sum_{k=0}^{\infty} \sum_{m=0}^k \frac{n^{2k} \Gamma\left(k+1, \frac{\gamma_0(1+n^2)}{\bar{\gamma}}\right)}{k! n!} \tag{9}$$

where $\Gamma(t, x) = \int_x^\infty \gamma^{t-1} e^{-\gamma} d\gamma$ is the upper incomplete Gamma function. As stated earlier γ_0 should satisfy the condition given in (4).

B. Constant transmitting power

Putting (8) into (5) and writing the confluent hypergeometric function in infinite series [12], the capacity for constant transmitting power techniques can be obtained as

$$C_{ora} = \frac{B(1+n^2)}{\bar{\gamma}} e^{-n^2} \sum_{k=0}^{\infty} \frac{n^{2k}}{k!^2} \left(\frac{(1+n^2)}{\bar{\gamma}}\right)^k \times \log_2 e \left[k! e^{\frac{(1+n^2)}{\bar{\gamma}}} \sum_{k=1}^n \frac{\Gamma\left(-1, \frac{(1+n^2)}{\bar{\gamma}}\right)}{\left(\frac{(1+n^2)}{\bar{\gamma}}\right)^k} \right] \tag{10}$$

C. Channel inversion with fixed rate

The capacity for this scheme requires a solution to the integral in R_{cifr} in (6). It can be solved by putting the PDF expression from (8) and then solving the resulting integral using [12, (7.621.4)]. The final expression after algebraic manipulation and simplification can be given as

$$R_{cifr} = \frac{(1+n^2)}{n\bar{\gamma}} e^{-n^2} \frac{\Gamma(0.00001)}{\Gamma(1)} e^{\frac{n^2}{2}} M_{1/2,0}(n^2) \tag{11}$$

Where $M_{a,b}(\cdot)$ is the WhittakerM function. Thus, an expression for the capacity of this scheme can be obtained by putting (11) into (6).

D. Truncated channel inversion with fixed rate

The capacity for this scheme requires a solution to the integral in R_{tifr} and $P_{out}(\gamma_0)$ in (7). Using (8), R_{tifr} can be obtained by solving the resulting integral using [12, (3.381.3)]. The final expression after simplification can be given as

$$R_{tifr} = \frac{(1+n^2)}{\bar{\gamma}} e^{-n^2} \sum_{k=0}^{\infty} \frac{n^{2k}}{k!^2} \Gamma\left(k, \frac{1+n^2}{\bar{\gamma}} \gamma_0\right) \tag{12}$$

An expression for $P_{out}(\gamma_0)$ is given in as

$$P_{out} = e^{-n^2} \sum_{k=0}^{\infty} \frac{n^{2k}}{k!^2} \left[\Gamma(k+1) - \Gamma\left(k+1, \frac{1+n^2}{\bar{\gamma}} \gamma_{th}\right) \right]$$

Thus, a final expression for the capacity of this scheme can be obtained by putting (12) and (13) into (7).

5. NUMERICAL RESULTS AND DISCUSSION

Obtained expressions for capacity with different power and rate adaptation techniques have been numerically evaluated for different values of fading parameter and plotted for illustration. Capacity (per unit bandwidth) vs. $\bar{\gamma}$ of ORA scheme has been plotted in Fig. 2. It can be observed from the figure that the capacity increases with increase in fading parameter n . Increase in fading parameter n indicates more power on the dominant path of the rice fading channel. Hence, as expected capacity have been increased. The capacity vs. $\bar{\gamma}$ for CIFR and TIFR schemes has been plotted in Figs. 3 and 4, respectively. In both schemes observation is similar to ORA system. In the plot of TIFR scheme we assume $\gamma_0 = 2dB$.

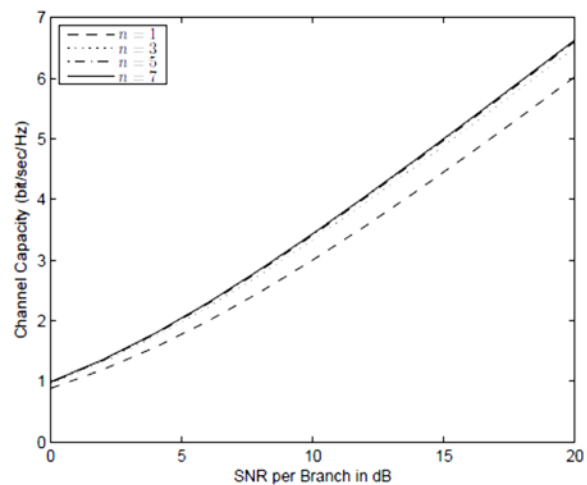


Fig. 2: Capacity of ORA scheme

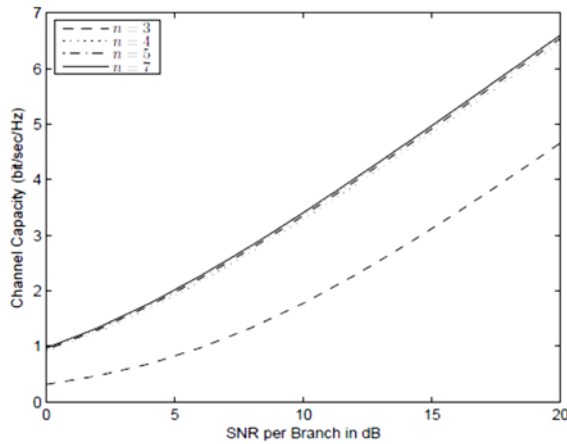


Fig. 3: Capacity of CIFR scheme

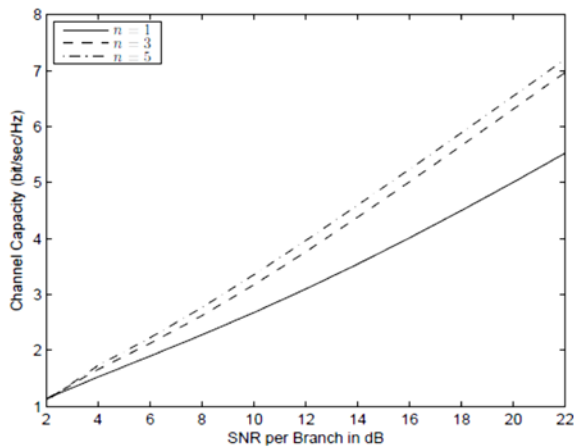


Fig. 4: Capacity of TIFR scheme

Therefore, plots are given for 2dB onwards. Capacity plots for OPRA scheme have not been included here, but it is possible to plot the capacity from the given analytical expression. Comparison of capacity schemes are given for different schemes considering fading parameter $n = 3$. As expected it has been observed that ORA perform best followed by TIFR and CIFR. It is due to reason that ORA is the best technique compared to the other two.

6. CONCLUSIONS

In this paper, we analyze the capacity of an adaptive system over rice fading channels, for different known power and rate adaptation transmission techniques. Numerically evaluated results have been plotted for different fading parameters. The obtained results are found in terms of incomplete Gamma function and the Whittaker function. These functions are easily available in mathematical softwares.

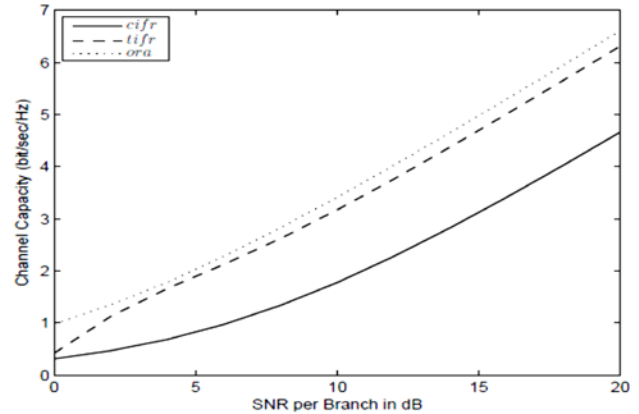


Fig. 5: Comparison of capacity schemes

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